

hep-th/0207065

Accelerating Universe without Bigbang singularity in Kalb-Ramond Cosmology

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Abstract

Existence of Bigbang singularity is considered to be the most serious drawback in the standard FRW cosmology. Furthermore to explain the accelerating phase of the Universe (recently detected experimentally) in such a model one needs to include a non-vanishing cosmological constant in the theory by hand. In this note we show that a string originated torsion in the background spacetime provides natural solutions to both these problems. The Universe evolving in a torsioned spacetime is not only free of Bigbang singularity but also exhibits acceleration during its evolution. The role of dilaton in this context is briefly discussed.

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Eversince Einstein-Cartan (EC) theory was proposed, spacetime *torsion* has provided a substantial amount of interest both in gravity and cosmology. In such a theory the coupling of electromagnetism with torsion has always been a problem because of the loss of the corresponding electromagnetic gauge invariance. Moreover the theory is faced with serious difficulties in absence of any confirmed experimental signature in favour of torsion. There was a revival in the interest for such a theory after the development of String theory where the massless second-rank antisymmetric Kalb-Ramond (KR) field $B_{\mu\nu}$ [1] arising in the heterotic string spectrum has a natural explanation in the form of spacetime torsion. It has recently been shown [2] that in the string theoretic scenario, spacetime torsion can in fact be identified with KR field strength augmented with the Chern-Simons (CS) extension. Such an extension, which originates to cancel gauge anomaly in the context of string theory, plays the crucial role in restoring electromagnetic $U(1)$ gauge invariance which is normally lost in the standard EC framework [2]. This enables one to study the effect of torsion on the propagation of electromagnetic waves preserving the gauge invariance. Investigations have also been carried out in the context of theories with large extra dimensions, namely those of Arkani-Hamed–Dimopoulos–Dvali (ADD) [3] and Randall–Sundrum (RS) [4]. An interesting possibility in such models is that of torsion existing in the bulk together with gravity, while all the standard model fields are confined to a 3-brane. In order to study the effect of extra dimensions on the spacetime torsion in the context of RS scenario it has been shown by Mukhopadhyaya et.al. [5] that inspite of having the same status as gravity in the bulk, effects of massless torsion becomes heavily suppressed on the standard model brane, thus producing the illusion of a torsionless Universe. This explains why the detection of torsion has been eluding us for so long.

In recent years, it has already been shown that the presence of KR field in the background spacetime may lead to various interesting astrophysical/cosmological phenomena like cosmic optical activity, neutrino helicity flip, parity violation etc. This motivates us to address some of the important problems associated with the standard Friedmann-Robertson Walker (FRW) model in the light of KR cosmology. The most longstanding problem is the so-called ‘Bigbang singularity’ and the most recent one concerns the observational evidences from type Ia Supernova data [6, 7] showing *acceleration* at a late time (may even be at the present epoch !) during the course of evolution of the Universe. A considerable amount of work has been done in recent past in circumventing these problems in the framework of FRW cosmology or in alternative cosmological models. However, the models obeying the ‘cosmological principle’, i.e., the large scale homogeneity and isotropy of the Universe, are of more significance for obvious observational reasons. In the present work, we examine these cosmological problems by considering a homogeneous and isotropic cosmological model in a background spacetime with torsion (or, equivalently, stringy KR field).

According to [2], the torsion tensor $T_{\mu\nu\lambda}$, which is the antisymmetrization of the affine connection in the corresponding spacetime and is chosen to be antisymmetric in all its indices, satisfies the constraint equation

$$T_{\mu\nu\lambda} = \sqrt{\kappa} H_{\mu\nu\lambda}, \quad (1)$$

where $\kappa = 16\pi G/c^4$ is the gravitational coupling constant and the tensor $H_{\mu\nu\lambda}$ is the strength of the KR field, i.e., $\partial_{[\mu} B_{\nu\lambda]}$, plus the Chern-Simons (CS) term which keeps the corresponding quantum theory anomaly-free. However, the CS term contains a suppression factor of the order of Planck mass relative to $\partial_{[\mu} B_{\nu\lambda]}$ and therefore can safely be dropped from $H_{\mu\nu\lambda}$ in the present analysis. While dealing with gravity in the context of string theory, one has to include, along with the KR field, the massless scalar dilaton field ϕ as well. The corresponding four-dimensional

effective action in Einstein frame is expressed as

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{\kappa} \left(R(g) - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi \right) - \frac{e^{-2\phi}}{12} H_{\mu\nu\lambda} H^{\mu\nu\lambda} + e^{-\beta\phi} \mathcal{L}_m \right] \quad (2)$$

where $R(g)$ is the usual scalar curvature of Einstein gravity. The dimensionless parameter β determines the coupling of the dilaton with external matter fields whose Lagrangian density is given by \mathcal{L}_m . The equations of motion that can be obtained from the above action are given by

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \frac{1}{2} \left(\partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} \partial_\alpha \phi \partial^\alpha \phi \right) = \kappa \left[e^{-\beta\phi} T_{\mu\nu}^{(m)} - e^{-2\phi} T_{\mu\nu}^{(KR)} \right] \quad (3)$$

$$D_\mu \left(e^{-2\phi} H^{\mu\nu\lambda} \right) = 0 \quad (4)$$

$$D_\alpha D^\alpha \phi = -\kappa \left(\frac{e^{-2\phi}}{6} H_{\mu\nu\lambda} H^{\mu\nu\lambda} - \beta e^{-\beta\phi} \mathcal{L}_m \right) \quad (5)$$

where D_μ stands for the covariant derivative defined in terms of the usual Christoffel connection. $T_{\mu\nu}^{(m)}$ is the energy-momentum tensor corresponding to the background matter distribution which for our cosmological model is taken to be a perfect fluid, and $T_{\mu\nu}^{(KR)}$ is the analogous contribution due to the KR field:

$$T_{\mu\nu}^{(m)} = \frac{1}{2} \left[-p g_{\mu\nu} + (p + \rho c^2) u_\mu u_\nu \right] \quad (6)$$

$$T_{\mu\nu}^{(KR)} = \frac{1}{12} \left(3 H_{\alpha\beta\mu} H^{\alpha\beta}{}_\nu - \frac{1}{2} g_{\mu\nu} H_{\alpha\beta\gamma} H^{\alpha\beta\gamma} \right) \quad (7)$$

p and ρ being the pressure and energy-density respectively, and $u_\mu = (1, 0, 0, 0)$ is the hypersurface-orthogonal four-velocity vector. The tensor $H_{\mu\nu\lambda}$ being completely antisymmetric in all indices — a three-form — can be expressed as the *Hodge*-dual of an one-form h_σ : $H_{\mu\nu\lambda} = \epsilon_{\mu\nu\lambda\sigma} h^\sigma$. Now, in the string context a further restriction is imposed on the KR field strength, viz., it is related to the derivative of a pseudoscalar *axion* field ξ via a duality: $H_{\mu\nu\lambda} = \epsilon_{\mu\nu\lambda\sigma} \partial^\sigma \xi$. Thus $h_\alpha = \partial_\alpha \xi$, and henceforth in our subsequent discussions we shall replace the KR field by the axion ξ using this duality.

In order to have a large-scale homogeneous and isotropic cosmological model, we have to consider the standard FRW metric structure:

$$ds^2 = c^2 dt^2 - a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right] \quad (8)$$

$a(t)$ being the scale-factor and k is the curvature. It can now easily be shown that for this type of metric the consistency of the field equations immediately demands the space derivatives of both the dilaton ϕ and the axion ξ to be zero, i.e., they depend only on time. Moreover, remembering that $H_{\mu\nu\lambda} = \partial_{[\mu} B_{\nu\lambda]}$ one can readily obtain the Bianchi identity: $\epsilon^{\mu\nu\lambda\rho} \partial_\rho H_{\mu\nu\lambda} = 0$ which implies that the axion satisfies the equation $D_0 D^0 \xi = 0$ with solution

$$\partial_0 \xi = \frac{\alpha}{a^2(t)} \quad (9)$$

α being a parameter determining the strength of the axion. The field equations now reduce to

$$\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} = \frac{\dot{\phi}^2}{12} + \frac{8\pi G}{3c^2} \left(e^{-\beta\phi} \rho c^2 - \frac{e^{-2\phi}}{2} \frac{\alpha^2}{a^4} \right) \quad (10)$$

$$\frac{2\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} = -\frac{\dot{\phi}^2}{4} - \frac{8\pi G}{c^2} \left(e^{-\beta\phi} p - \frac{e^{-2\phi}}{2} \frac{\alpha^2}{a^4} \right) \quad (11)$$

$$\ddot{\phi} + \frac{2\dot{a}}{a} \dot{\phi} = \frac{8\pi G}{c^2} \left(2e^{-2\phi} \frac{\alpha^2}{a^4} + \beta e^{-\beta\phi} \rho c^2 \right) \quad (12)$$

where overhead dot stands for derivative with respect to time. Considering a general equation of state $p = \omega \rho c^2$, we obtain from the field equations the following set of equations

$$\frac{\dot{b}^2}{b^2} + \frac{\chi}{b^2} = \frac{\dot{\phi}^2}{12} + \frac{\lambda_0}{3} \left(e^{-\beta\phi} \zeta - \frac{e^{-2\phi}}{2} \frac{\sigma_0}{b^4} \right) \quad (13)$$

$$\frac{\ddot{b}}{b} = -\frac{\dot{\phi}^2}{6} + \frac{\lambda_0}{3} \left(e^{-2\phi} \frac{\sigma_0}{b^4} - \frac{3\omega + 1}{2} e^{-\beta\phi} \zeta \right) \quad (14)$$

$$\ddot{\phi} + \frac{2\dot{b}}{b} \dot{\phi} = \lambda_0 \left(2e^{-2\phi} \frac{\sigma_0}{b^4} + \beta e^{-\beta\phi} \zeta \right) \quad (15)$$

$$\frac{d}{db}(\zeta b^3) = -3\omega \zeta b^2 + \frac{\beta}{2}(\zeta b^3) - e^{(\beta-2)\phi} \frac{\sigma_0}{b^2} \left(2b \frac{d\phi}{db} - 1 \right) - \frac{b\dot{\phi}^2}{2\lambda_0} e^{\beta\phi} \quad (16)$$

where, for convenience, we have used dimensionless quantities $a/a_0 = b$, $\rho/\rho_0 = \zeta$ and the notation $\chi = kc^2/a_0^2$, $\lambda_0 = 8\pi G\rho_0$ and $\sigma_0 = (\alpha^2 a_0^{-4})/(\rho_0 c^2)$. The subscript 0 generally stands for the values of the quantities at the present epoch. Recalling that the KR field strength, being expressed in terms of the axion, is given by $\partial_0 \xi = \alpha a^{-2}(t)$, the quantity σ_0 defined above gives a relative measure of the present values of KR field and effective energy density of the Universe.

In our low-energy effective theory, we presently explore the effect of only axion on the evolution of the Universe and assume that the dilaton has frozen to a particular Vacuum Expectation Value (VEV) much earlier. In such a case the field equations simplify to

$$\frac{\dot{b}^2}{b^2} + \frac{\chi}{b^2} = \frac{\lambda_0}{3} \left(\zeta - \frac{\sigma_0}{b^4} \right) \quad (17)$$

$$\frac{\ddot{b}}{b} = \frac{\lambda_0}{3} \left(\frac{\sigma_0}{b^4} - \frac{3\omega + 1}{2} \zeta \right) \quad (18)$$

$$\frac{d}{db}(\zeta b^3) = -3\omega \zeta b^2 + \frac{\sigma_0}{b^2}. \quad (19)$$

We now seek specific solutions of these equations separately for the present matter-dominated (dust) Universe and for the early radiation-dominated Universe.

1 Matter-dominated Universe

Presently, in the Universe $p \ll \rho c^2$, i.e., effectively $\omega \approx 0$. Eq.(19) can be solved to obtain

$$\zeta = \frac{1 + \sigma_0}{b^3} - \frac{\sigma_0}{b^4}, \quad (20)$$

which when plugged into Eqs.(17) and (18) yields

$$\frac{\dot{b}^2}{b^2} + \frac{\chi}{b^2} = \frac{\lambda_0}{3} \left(\frac{1 + \sigma_0}{b^3} - \frac{3\sigma_0}{2b^4} \right) \quad (21)$$

$$\frac{\ddot{b}}{b} = \frac{\lambda_0}{6} \left(\frac{1 + \sigma_0}{b^3} - \frac{3\sigma_0}{b^4} \right). \quad (22)$$

These equations produce specific bounds on the scale-factor b in the following two cases:

(i) For a real KR field, we have $\sigma_0 > 0$, i.e., a real and positive ratio of the present values of the KR field ‘axion’ and the effective energy density. Now, since both energy density $\zeta (= \rho/\rho_0)$ and scale-factor $b (= a/a_0)$ have to be non-negative quantities, Eq.(20) sets a lower bound on b : $b \geq \sigma_0/(1 + \sigma_0)$. The limiting equality ($b = 0$ and hence a singular ζ) can be attained only for $\sigma_0 = 0$ which corresponds to the usual FRW model. But whenever the KR field is present ($\sigma_0 > 0$), the scale factor b is always bound to remain positive and the energy density ζ finite — a possibility of avoiding the so-called ‘Bigbang singularity’. In fact, as we backtrace the model to the epoch $t = 0$, we find that the repulsive action effected by torsion (that is induced by the KR field axion) poses a compelling requirement that the Universe must have started with a finite size and density.

If now we consider the criterion that the Universe passes through a phase of accelerating expansion during the course of its evolution, i.e., $\ddot{b}(t) > 0$ for a particular t , it follows from Eq.(22) that an upper limit is also imposed on b : $b < 3\sigma_0/(1 + \sigma_0)$. This restriction is perfectly consistent with the other bound on b mentioned above and shows that a real KR field can actually produce such acceleration. Moreover, if the Universe is supposed to be accelerating presently, i.e., for $b = a/a_0 = 1$, then we further have the condition $\sigma_0 > 1/2$ which gives an estimate of how intense the KR field is relative to the present energy density.

(ii) For imaginary KR field, $\sigma_0 < 0$, and we write $\sigma_0 = -\bar{\sigma}_0$, ($\bar{\sigma}_0$ a positive quantity). In this case the condition that ζ and b must have to be non-negative sets an upper bound on b : $b \leq \bar{\sigma}_0/(\bar{\sigma}_0 - 1)$ with $\bar{\sigma}_0 > 1$, wherefrom it cannot be ascertained whether or not the Bigbang singularity is avoided. The further criterion of having an accelerating Universe puts a rather absurd lower bound on b : $b > 3\bar{\sigma}_0/(\bar{\sigma}_0 - 1)$, completely inconsistent with the more stringent upper bound given above. Thus acceleration is not possible in an imaginary KR background, at any phase during the course of evolution of the Universe. A cosmological model can, however, exist for such an imaginary KR field axion where the latter is stronger than the present matter density since $\bar{\sigma}_0 > 1$. But, if the observational evidences regarding a late-time accelerating Universe is considered, such a model may not be that interesting. As acceleration is a major issue in building up a cosmological model in the present work, we concentrate here only on the case of a real KR field.

Now, by solving Eq.(21) for a real KR field, we obtain the following sets of parametric solutions in the three separate cases:

(a) Zero Curvature ($k = 0$ and hence $\chi = kc^2/a_0^2 = 0$) [Spatially-flat Universe]:

$$\begin{aligned} b(\eta) &= \frac{\lambda_0}{12}(1 + \sigma_0) \eta^2 + \frac{3\sigma_0}{1 + \sigma_0} \\ t(\eta) &= \frac{\lambda_0}{36}(1 + \sigma_0) \eta^3 + \frac{3\sigma_0}{1 + \sigma_0} \eta. \end{aligned} \quad (23)$$

(b) Constant Positive Curvature ($k = 1$ and hence $\chi > 0$) [Closed Universe]:

$$\begin{aligned} b(\eta) &= \frac{\lambda_0(1 + \sigma_0)}{6\chi} \left[1 - \sqrt{1 - \frac{36\chi\sigma_0}{\lambda_0(1 + \sigma_0)^2}} \cos(\eta\sqrt{\chi}) \right] \\ t(\eta) &= \frac{\lambda_0(1 + \sigma_0)}{6\chi} \left[\eta - \sqrt{1 - \frac{36\chi\sigma_0}{\lambda_0(1 + \sigma_0)^2}} \frac{\sin(\eta\sqrt{\chi})}{\sqrt{\chi}} \right]. \end{aligned} \quad (24)$$

(c) Constant Negative Curvature ($k = -1$ and hence $\chi < 0$) [Open Universe]:

$$b(\eta) = \frac{\lambda_0(1 + \sigma_0)}{6|\chi|} \left[\sqrt{1 + \frac{36\chi\sigma_0}{\lambda_0(1 + \sigma_0)^2}} \cosh(\eta\sqrt{|\chi|}) - 1 \right]$$

$$t(\eta) = \frac{\lambda_0(1 + \sigma_0)}{6|\chi|} \left[\sqrt{1 + \frac{36\chi\sigma_0}{\lambda_0(1 + \sigma_0)^2}} \frac{\sinh(\eta\sqrt{|\chi|})}{\sqrt{|\chi|}} - \eta \right]. \quad (25)$$

These results at once reduce to the corresponding FRW forms as soon as we put $\sigma_0 = 0$. Characteristic graphs of time against the scale factor b and also against the observationally significant quantities, viz., the Hubble parameter $H = \dot{b}/b$ and the deceleration parameter $q = -\frac{1}{H^2}(\ddot{b}/b)$, have been plotted in Figs.(1) - (4) which depict the evolution of the matter-dominated Universe in a KR background for various values of curvature χ . The parameter σ_0 has been assigned a typical value $1/2$ in Fig.(1) to show the nature of the b vs t curves for flat, closed and open Universe, while in Figs.(2) and (3) the $b-t$ and $H-t$ plots are shown characteristically for a flat Universe for various values of σ_0 . The time-scale is, as usual, taken to be the inverse of the Hubble constant $H_0 = \dot{b}(t_0)$, experimentally measured to be $\sim 10^{-18}s^{-1}$. For positive σ_0 , both the features of no singularity at the origin $t = 0$ and accelerating Universe are clear from these graphs. Acceleration at a certain stage in the evolution of the Universe can clearly be observed in the $q-t$ plots in Figs.(4a) - (4c), where the additional feature of present-day acceleration for $\sigma_0 > 1/2$ is prominent in all the cases, viz., flat, closed and open.

2 Radiation-dominated Universe

Extrapolation of the matter-dominated model to the epoch $t = 0$ to address the problem of Bigbang singularity may not be very reliable because of the radiation dominance at the early ages. We now therefore study a model of radiation-dominated Universe in a KR background. In such a model we have $p = \frac{1}{3}\rho c^2$, i.e., $\omega = 1/3$. We obtain from Eqs.(17)-(19) that

$$\zeta = \frac{1}{b^4}(1 + \sigma_0 \ln b) \quad (26)$$

$$\frac{\dot{b}^2}{b^2} + \frac{\chi}{b^2} = \frac{\lambda_0}{3b^4} \left(1 - \frac{\sigma_0}{2} + \sigma_0 \ln b \right) \quad (27)$$

$$\frac{\ddot{b}}{b} = -\frac{\lambda_0}{3b^4} (1 - \sigma_0 + \sigma_0 \ln b). \quad (28)$$

Once again we find specific bounds on the scale factor b in the cases of real and imaginary KR fields. For real KR field, i.e., $\sigma_0 > 0$, it is evident from Eq.(26) that to keep ζ and b non-negative we must have $b \geq e^{-1/\sigma_0}$ which clearly shows no sign of a singularity at Bigbang if we backtrace this radiation model to the epoch $t = 0$. In an imaginary KR background, $\sigma_0 = -\bar{\sigma}_0 < 0$, and non-negative ζ and b only gives an upper bound on b : $b \leq e^{1/\bar{\sigma}_0}$ wherefrom no inference can be drawn regarding Bigbang singularity. Moreover, similar to the case for the matter-dominated model, here also it is easy to show from Eq.(28) that the other bound posed on b by the requirement of acceleration at any epoch, is feasible only if the KR field is real. Hence, for a radiation-dominated model also, we disregard the case of an imaginary KR field.

As radiation-domination in the Universe is supposed to be prevalent in the early stages of its evolution, we can ignore, without appreciable error, the curvature χ . Moreover, in absence of an acceptable exact solution, we adopt a perturbative approach where a small KR field perturbs the standard FRW background. A small KR field signifies the parameter $\sigma_0 \ll 1$, whence we obtain the solution for b :

$$[b(t)]^2 = \sqrt{\frac{\lambda_0}{3}} \cdot t \left[2 + \frac{\sigma_0}{2} (\ln t - 1) + \frac{\sigma_0^2}{16} \{ (\ln t)^2 - 4 \ln t + 5 \} + O(\sigma_0^3) \right] + O(\lambda_0) \quad (29)$$

$\lambda_0 = 8\pi G\rho_0/3$ itself being a very small quantity as both $G \sim (\text{Planck mass})^{-2}$ and the present radiation density ρ_0 are really small. All the arbitrary constants appearing in the solution are present in $O(\lambda_0)$ and further higher terms, and manifest themselves in a non-zero scale-factor b at the Bigbang epoch $t = 0$.

Some renewed study of the Supernova Ia data provides evidence of not only a late-time accelerating Universe but a decelerating Universe in the remote past as well [8]. As far as the analysis in the present work is concerned, both the radiation and matter-dominated models are quite simplified and leads to a fair bit of approximation especially while backtracing to the early stages of the Universe. A rather rigorous theory takes into account, alongwith the KR-axion, the effect of the dilaton field as well. The presence of the dilaton ϕ yields the highly coupled field equations (13) - (16) which are very difficult to solve analytically. However, as can easily be read from Eq.(14), the dilaton causes a negative effect on the acceleration, viz, the term $-\dot{\phi}^2$ in Eq.(14), and hence produces an opposite effect to that of the axion. This may, in turn, be the reason behind a past decelerating phase of the Universe. A more exact determination of this may be done by solving equations (13) - (16) numerically.

In conclusion, we have found that the axion(dual to the KR field strength) in a stringy cosmological model provides a natural solution to the problem of Bigbang singularity present in the FRW model. Moreover such an axion causes the universe to pass through an accelerating phase which at least qualitatively explains the recent experimental findings through the Supernova Ia. The other scalar massless mode of String theory, namely the dilaton, is also found to play a crucial role to control the expansion rate at various epochs. All these may lead to an indirect support for a string inspired KR cosmology once a more quantitative estimation is done. However through this work we propose a possible resolution of two of the most important problems prevailing in the present cosmological scenario.

Acknowledgment

This work is supported by Project grant no. 98/37/16/BRNS cell/676 from The Board of Research in Nuclear Sciences, Department of Atomic Energy, Government of India, and the Council of Scientific and Industrial Research, Government of India.

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